

# Bridge Measurement Analysis

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## Introduction

At the United States Academy there are several engineering majors, including Systems Engineering. This program offers excellent systems integration education. In particular the major concentrates on control of electrical, computer and mechanical systems. In addition to several tracks, students have the opportunity to independently research a field of interest. This is a great opportunity for teachers and students to pursue more in-depth analyses. This paper will describe one such experiment in the field of metrology.

Very often engineering laboratories at undergraduate schools are well equipped with power supplies, signal generators, oscilloscopes and general-purpose multimeters. This set allows teachers and students to set up test-beds for most of the basic electronics circuits studied in different engineering tracks. Modern instrumentation is in general user-friendly and students like using the equipment. However, students are often not aware that there are two pieces of information necessary to establish a measurement result: the numerical value of the measured quantity and the uncertainty with which that measurement was performed. In order to achieve high measurement accuracy, more complex measurement systems must be developed. This paper will describe the process of analyzing a bridge measurement using MATLAB<sup>‡</sup>.

## Measurement Bridge

One of the basic circuits that demonstrate the concept of a current/voltage divider is a Wheatstone bridge (given in Figure 1.) A source voltage is applied to a parallel connection of impedances. The source current is divided into a branch with impedances  $Z_1$  and  $Z_2$  and a branch with impedances  $Z_3$  and  $Z_4$ . The current flows through the detector when the bridge is not balanced. In order for the student to understand the relationship between the current through the detector and the other elements in the bridge, it is beneficial to calculate the current through the detector. Electrical circuit analysis may be performed using symbolic functions in MATLAB.

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<sup>‡</sup> In order to describe the procedures discussed in this paper, commercial products are identified. In no case does such identification imply recommendation or endorsement by the National Institute of Standards and Technology or that the materials or equipment specified are necessarily the best available for the purpose.

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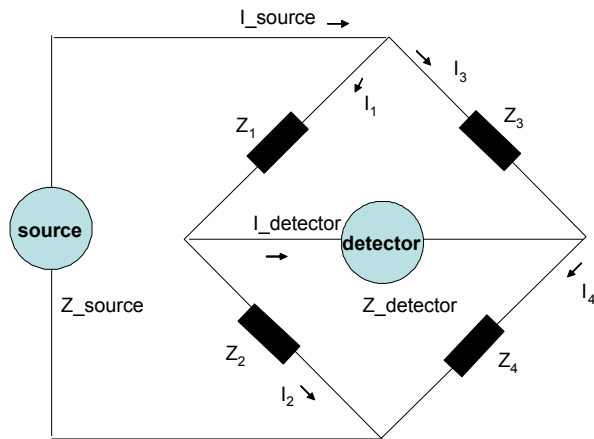


Figure 1. Wheatstone bridge

MATLAB script 1 file:

```
~~~~~
%Declaring symbolic variables
syms Z1 Z2 Z3 Z4 Z_detector V_source Z_source Circuit_matrix Voltage_vector real

% Current_vector = [I1      I2      I3      I4      I_detector      I_source];

Circuit_matrix = [ Z1      Z2      0      0      0      Z_source;
                   0      0      Z3      Z4      0      Z_source;
                   -1      0      -1      0      0      1;
                   0      1      0      1      0      -1;
                   1      -1      0      0      -1      0;
                   0      -Z2      0      Z4      Z_detector 0];

Voltage_vector = [ V_source  V_source  0      0      0      0]';

Current_vector = inv(Circuit_matrix)*Voltage_vector;
~~~~~
```

This script file uses Kirchhoff voltage and current law to compute the current through the detector:

$$I_{\text{detector}} = \frac{(Z_3 Z_2 - Z_4 Z_1) V_{\text{source}}}{Z_3 Z_{\text{detector}} Z_1 + Z_3 Z_{\text{detector}} Z_2 + Z_1 Z_4 Z_3 + Z_3 Z_2 Z_4 + Z_3 Z_2 Z_1 + Z_1 Z_4 Z_2 + Z_{\text{detector}} Z_4 Z_1 + Z_{\text{detector}} Z_4 Z_2}$$

The bridge is "balanced" when there is no current through the detector. This condition is accomplished when  $Z_3 Z_2 = Z_4 Z_1$ . Practically, this means that we can measure the unknown impedance,  $Z_1$ , by adjusting the other impedances in the bridge to meet the balancing condition,  $Z_1 = Z_3 Z_2 / Z_4$ .

The bridge balance derivation is carried through using impedances in order to introduce general principles.

### **Multimeter Measurements**

Let us assume that we want to measure resistance. The reason for this assumption is that it is rather easy to build a resistive Wheatstone bridge. Also it is instructional to compare bridge measurements to the result obtained with an ohmmeter. Simple hand-held multimeters are readily available in laboratories. The instruments that measure impedance (LCR meters) are expensive and not broadly available in classroom laboratories.

A hand-held ohmmeter will give us an instant result with resolution of 2-3 digits within the manufacturer's specified range. This is a very convenient process and is suitable for many student-oriented applications. We can easily answer the question of what is the numerical value of the measured quantity. But we need to also ask the questions regarding the uncertainty with which we have performed this measurement and whether or not the accuracy is satisfactory for our application. To answer those questions, students must obtain the meter manufacturer's error specifications. Usually the uncertainty of a good student version digital instrument is at the order of its least significant digit. This value could be at the percent level. The reason for such a high level of uncertainty relates to the physical realization of the measuring process. Often a battery is used to produce known current through the unknown resistor. Voltage drop is measured across the unknown resistor to establish its resistance value. Due to many environmental influences this process is not very stable and precise. Frequently, manufacturers select to reduce resolution in order to have repeatable measurements.

The conclusion of this exercise is that digital multimeters provide quick and simple measurements and that special attention is required to interpret the accuracy of the result relative to the application. Students tend to believe that the digital result is absolutely accurate. But the skill to evaluate the result comparatively to the other components of the project has to be developed further among many of the students.

### **Bridge Measurements**

Let us now consider bridge measurements. The bridge must be constructed using a voltage source, a detector and a set of resistors. Practically, we can set the experiment in such a way that two resistors,  $R_2$  and  $R_4$ , are used to set the range. So their ratio is relevant to the measurement process. One resistor,  $R_3$ , has to have very good resolution in order to fine tune the result.

In the case when complex impedance must be measured, bridge components should be variable impedances. The bridge should have isolation transformers in order to maintain a proper ground connection. Also the voltage source and detector must be alternating current devices. But probably the most complex change is the process of balancing the bridge. Both in-phase and quadrature components of current must be driven to zero. This condition allows for measuring the real and imaginary components of impedance. To cover this case, a new experiment should be constructed.

In general, a bridge measurement is a much more complex experiment than connecting an unknown resistance to a multimeter. But let us consider the benefits from such an exercise.

1. Students acquire hands-on experience in connecting different instruments. This is a valuable skill for systems engineers since their main job is modular integration.
2. The circuit is very simple so the students can fully understand it and analyze it in detail. This is absolutely necessary in order to be able to improve the experiment. Teaching an iterative design process that converges into a good project is a critical component of a high-quality college engineering program.
3. The measurement result does not depend on the value of the voltage source. Only its short-term stability is needed to perform the balance.
4. The sensitivity of the detector plays the role in determining the balance but keep in mind that we just have to confirm zero current. Many instruments readily available have a very reliable zero determination (most notably lock-in amplifiers).
5. Also, we should be able to agree that it is not difficult to calculate the value of the unknown resistor using the bridge balance formula.
6. The measurement result depends only on the set of resistors. Those are passive components so it is easier to define their own precision. Usually the practical limitation of this experiment is the accuracy, resolution and stability of the resistor  $R_3$ .

Furthermore, we have to answer the question about the uncertainty of the bridge measurement. We will discuss two types of errors involved. Engineering seniors are familiar with the statistics and can comprehend that by repeating the measurements, we can lower the random errors straightforwardly.

But random errors are not the only types of errors so we need to consider the systematic errors as well. Systematic errors depend on the instrumentation used in the particular measuring process and are a function of the physical principle involved in measuring the unknown value.

The measured value,  $Z_1$ , depends on  $Z_3$ ,  $Z_2$  and  $Z_4$ , and the relationship is given as  $Z_1 = Z_3 Z_2 / Z_4$ . We need to find the partial derivatives for each contributing factor:

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta Z_3}{Z_3} ; \quad \frac{\Delta Z_1}{Z_1} = \frac{\Delta Z_2}{Z_2} ; \quad \frac{\Delta Z_1}{Z_1} = - \frac{\Delta Z_4}{Z_4} ,$$

where  $\Delta$  is the uncertainty with which we know a particular value. To get the overall uncertainty we have to calculate the worst-case scenario:

$$\frac{\Delta Z_1}{Z_1} = \left| \frac{\Delta Z_3}{Z_3} \right| + \left| \frac{\Delta Z_2}{Z_2} \right| + \left| \frac{\Delta Z_4}{Z_4} \right| .$$

This equation gives us the formula to calculate the uncertainty with which we know our measurements. It is given as a relative error. The resistors that we use to build simple electronic circuits have tolerances at the percent level. This means that the uncertainty of the result will be three times the uncertainty with which we know the bridge components.

Now we also have to figure out what factors influence the bridge balance. Let us analyze in more detail the detector current derivation. To find out what is the smallest change of unknown resistance that can be observed, we have to know the sensitivity of the detector, meaning the smallest deflection possible to see.

Let us use notation  $Z_1$  for the measured value and its value at the balance  $Z_{1\text{balance}} = Z_3 Z_2 / Z_4$ . Then the bridge sensitivity relative to the measured value [1] is:

$$\frac{\Delta I_{\text{detector}}}{\Delta Z_1} = \frac{-Z_4 V_{\text{source}}}{Z_3 Z_2 Z_1 + Z_1 Z_4 Z_3 + Z_1 Z_4 Z_2 + Z_3 Z_2 Z_4 + Z_{\text{source}} (Z_3 Z_2 + Z_1 Z_4 + Z_4 Z_3 + Z_2 Z_1) + Z_{\text{detector}} [Z_1 Z_3 + Z_3 Z_2 + Z_4 Z_1 + Z_4 Z_2 + Z_{\text{source}} (Z_3 + Z_2 + Z_4 + Z_1)]}$$

MATLAB script that calculates the sensitivity of the bridge is given.

MATLAB script2 file:

```
~~~~~

%This script is the continuation of the script1.

Z2=10;Z4=10;Z3=10;Zsource=1;Zdetector=100;Vsource=20;
dIdetector=0.1e-3;      %smallest detector deflection

Z1=Z2*Z3/Z4 % bridge balance

% S=bridge sensitivity  S=dI_detector/dZ1, substitute Z1=Z2*Z3/Z4

S=-Z4*V_source/
(Z3*Z2*Z1+Z1*Z4*Z3+Z1*Z4*Z2+Z3*Z2*Z4+Z_source*(Z3*Z2+Z1*Z4+Z4*Z3+Z2*Z1)
+Z_detector*(Z1*Z3+Z3*Z2+Z4*Z1+Z4*Z2+Z_source*(Z1+Z2+Z3+Z4)));
dz1=dIdetector/S      %bridge resolution

~~~~~
```

Let us consider the following bridge components  $V_{\text{source}}=20 \text{ V}$ ,  $Z_{\text{source}}=1 \Omega$ ,  $Z_{\text{detector}}=100 \Omega$ ,  $Z_2=10 \Omega$ ,  $Z_4=10 \Omega$ , and  $Z_3=10 \Omega$ . Also let us assume that the smallest deflection that we can see on the detector is  $\Delta I_{\text{detector}}=0.1 \text{ mA}$ . Then we can use MATLAB to calculate measured value at balance,  $Z_1$ , the bridge sensitivity  $S$  and bridge resolution,  $\Delta Z_1$  given the smallest detector deflection,  $\Delta I_{\text{detector}}$ .

The result from this exercise is that the measured value at balance,  $Z_1=10\Omega$ , the bridge sensitivity  $S=4 \text{ mA}/\Omega$  and bridge resolution,  $\Delta Z_1= 0.02 \Omega$ .

It is very important to understand the difference between the resolution and the uncertainty. Let us assume that bridge resistors  $Z_2$ ,  $Z_3$  and  $Z_4$  have manufacturer's specified tolerances of 0.1%. In other words, the uncertainty with which we know the bridge resistors is 0.1%. This translates to the fact that we measured a  $10 \Omega$  resistor with uncertainty of 0.3%  $\Rightarrow Z_1=10 \pm 0.03 \Omega$ . Note that using our bridge we could have observed the detector deflection for the resistance change of  $0.02 \Omega$ .

The numerical values for this exercise were chosen to reflect maximum bridge resolution. We can easily build a routine that can find resolution for different bridge settings as shown in MATLAB script3.m. This routine also selects parameters for the best bridge resolution.

MATLAB script3 file:

```
~~~~~  
% This is the continuation of script2.m  
m=1;  
for Z2=10:100:10000;  
    for Z4=1:1:10;  
        for Z3=1:0.1:10;  
            Z1=Z2*Z3/Z4; Z1m(m)=Z1;  
            S=-Z4*V_source/  
            (Z3*Z2*Z1+Z1*Z4*Z3+Z1*Z4*Z2+Z3*Z2*Z4+Z_source*(Z3*Z2+Z1*Z4+Z4*Z3+Z2*Z1)+  
            Z_detector*(Z1*Z3+Z3*Z2+Z4*Z1+Z4*Z2+Z_source*(Z1+Z2+Z3+Z4)));  
  
            dr=di/S; sm(m)=S; m=m+1;  
  
            if abs(dr)/Z1<drmin; drmin=abs(dr)/Z1; z20=Z2; z40=Z4; z30=Z3; s0=S; z10=Z1; end  
                end  
        end  
    end  
  
    loglog(Z1m, di./Z1m./S, 'b.')
```

~~~~~

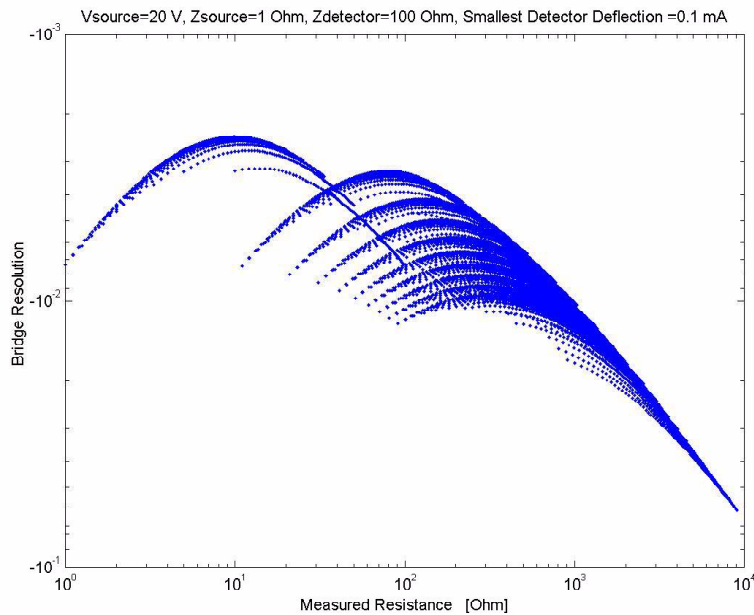


Figure 2. Bridge resolution as a function of measured resistance.

Figure 2 shows how the bridge resolution depends on the measured value. It is important to notice that bridge sensitivity and resolution are related and depend not only on measured value, but also on the characteristics of the instrumentation used (namely input impedance of the detector and bridge drive voltage)

## Conclusion

This type of exercises provides the students with valuable project experience. The circuit analyzed is simple enough so that it can be analyzed in detail. Once we have MATLAB routines for bridge balance and sensitivity derivations it is not difficult to see that we can explore different bridge settings and use different instrumentation to determine balance conditions.

Students respond very well to lessons in which they can construct an experiment. It has been the instructor's experience that the most valuable lessons are those in which students are able to fully comprehend the analysis and then design a test bed. Metrology experiments are a good tool to introduce system analysis and design.

#### References :

- [1] Bego V. "Electrical Measurements", (in Serbo-Croatian) , Tehnicka knjiga-Zagreb, Third edition, 1975.